Pensieve Header: Wheeled Semi-Symmetrized calculus in the 2D quotient: Solving for the “conj” coefficients, III.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\w-Computations"];
<< "Wheeled Semi-Symmetrized 2D Calculus.m"
```

- **conj2-3 are designed to work only in the feedback case.**

```
conj2[y_, x_][μ_] := Module[
   {γ, ξc},
   ξc = D[μ, h[x]] /. t[s_] :> c[s];
   γ = Coefficient[μ, ar[y, x]];
   μCollect[μ + W[γ, ξc]]
  ];
```

```
μ3 = α1 ar[1, 1] + α3 ar[1, 2] + α4 ar[1, 3];
μ3 = α1 ar[1, 1] + α2 ar[2, 1] + α3 ar[1, 2] + α4 ar[1, 3];
Riffle[
 ComposeList[
   ops = {conj2[1, 3], conj2[1, 2], hm[2, 3, 2]},
   μ3 = α1 ar[1, 1] + 0 α2 ar[2, 1] + α3 ar[1, 2] + α4 ar[1, 3]
  ] // μForm,
 ops
]
```

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix}, \text{conj2}[1,3], \begin{pmatrix} W[\alpha_4, c[1]\alpha_4] & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix}, \text{conj2}[1,2], \begin{pmatrix} W[\alpha_3, c[1]\alpha_3] + W[\alpha_4, c[1]\alpha_4] & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix}, \text{hm}[2,3,2], \begin{pmatrix} W[\alpha_3, c[1]\alpha_3] + W[\alpha_4, c[1]\alpha_4] & h[1] & h[2] \\ t[1] & \alpha_1 & \alpha_4 + \alpha_3 (1 + c[1]\alpha_4) \end{pmatrix}\right\}$$

```
Riffle[
 ComposeList[
   ops = {hm[2, 3, 2], conj2[1, 2]},
   μ3
  ] // μForm,
 ops
]
```

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix}, \text{hm}[2,3,2], \begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & \alpha_1 & \alpha_4 + \alpha_3 (1 + c[1]\alpha_4) \end{pmatrix}, \text{conj2}[1,2], \begin{pmatrix} W[\alpha_4 + \alpha_3 (1 + c[1]\alpha_4), c[1](\alpha_4 + \alpha_3 (1 + c[1]\alpha_4))] & h[1] & h[2] \\ t[1] & \alpha_1 & \alpha_4 + \alpha_3 (1 + c[1]\alpha_4) \end{pmatrix}\right\}$$

```
{c2l = μ3 // conj2[1, 2] // conj2[1, 3] // hm[2, 3, 2],
 c2r = μ3 // hm[2, 3, 2] // conj2[1, 2]}
```

$$\{h[2](\alpha_4 + \alpha_3 (1 + c[1]\alpha_4))\, t[1] + h[1](\alpha_1 t[1] + \alpha_2 t[2]) + W[\alpha_3, c[1]\alpha_3] + W[\alpha_4, c[1]\alpha_4],$$
$$h[2](\alpha_4 + \alpha_3 (1 + c[1]\alpha_4))\, t[1] + h[1](\alpha_1 t[1] + \alpha_2 t[2]) + W[\alpha_4 + \alpha_3 (1 + c[1]\alpha_4), c[1](\alpha_4 + \alpha_3 (1 + c[1]\alpha_4))]\}$$

```
c2l - c2r // FullSimplify
```

$$-W\,(-1 + \text{Log}[1 + c[1]\alpha_3] + \text{Log}[1 + c[1]\alpha_4] - \text{Log}[(1 + c[1]\alpha_3)(1 + c[1]\alpha_4)])$$

```
sols = SolveAlways[c2l == c2r, {t[1], t[2], h[1], h[2], α1, α2, α3, α4, c[1], W}]
```

```
{{Log[1 + c[1] α4] → 0}, {Log[1 + c[1] α4] → 1},
 {Log[1 + c[1] α4] → Log[(1 + c[1] α3) (1 + c[1] α4)]},
 {Log[(1 + c[1] α3) (1 + c[1] α4)] → 0}, {}, {}, {}}
```

```
W (a1 + a2 c[y]) γ / (1 + (a3 + a4 c[y]) γ)
```

$$\frac{W\,\gamma\,(a1 + a2\,c[y])}{1 + \gamma\,(a3 + a4\,c[y])}$$

```
(W (a1 + a2 c[y]) γ / (1 + (a3 + a4 c[y] γ))) /. sols
```

```
{0}
```

■ conj3

```
conj3[y_, x_][μ_] := Module[
   {γ, ξc},
   γ = Coefficient[μ, ar[y, x]];
   μCollect[μ + W Log[c[y] γ + 1]]
  ];
```

```
μ3 = α1 ar[1, 1] + α3 ar[1, 2] + α4 ar[1, 3];
μ3 = α1 ar[1, 1] + α2 ar[2, 1] + α3 ar[1, 2] + α4 ar[1, 3];
Riffle[
 ComposeList[
   ops = {conj3[1, 3], conj3[1, 2], hm[2, 3, 2]},
   μ3 = α1 ar[1, 1] + 0 α2 ar[2, 1] + α3 ar[1, 2] + α4 ar[1, 3]
  ] // μForm,
 ops
]
```

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix}, \text{conj3}[1, 3], \begin{pmatrix} W\,\text{Log}[1 + c[1]\,\alpha_4] & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix},\right.$$
$$\text{conj3}[1, 2], \begin{pmatrix} W\,(\text{Log}[1 + c[1]\,\alpha_3] + \text{Log}[1 + c[1]\,\alpha_4]) & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix},$$
$$\left.\text{hm}[2, 3, 2], \begin{pmatrix} W\,(\text{Log}[1 + c[1]\,\alpha_3] + \text{Log}[1 + c[1]\,\alpha_4]) & h[1] & h[2] \\ t[1] & \alpha_1 & \alpha_4 + \alpha_3\,(1 + c[1]\,\alpha_4) \end{pmatrix}\right\}$$

```
Riffle[
 ComposeList[
   ops = {hm[2, 3, 2], conj3[1, 2]},
   μ3
  ] // μForm,
 ops
]
```

$$\left\{\begin{pmatrix} 0 & h[1] & h[2] & h[3] \\ t[1] & \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix}, \text{hm}[2, 3, 2], \begin{pmatrix} 0 & h[1] & h[2] \\ t[1] & \alpha_1 & \alpha_4 + \alpha_3\,(1 + c[1]\,\alpha_4) \end{pmatrix},\right.$$
$$\left.\text{conj3}[1, 2], \begin{pmatrix} W\,\text{Log}[(1 + c[1]\,\alpha_3)\,(1 + c[1]\,\alpha_4)] & h[1] & h[2] \\ t[1] & \alpha_1 & \alpha_4 + \alpha_3\,(1 + c[1]\,\alpha_4) \end{pmatrix}\right\}$$

```
Simplify[
 (W (Log[1 + c[1] α3] + Log[1 + c[1] α4]) == W Log[(1 + c[1] α3) (1 + c[1] α4)]) /.
  Log[a_] + Log[b_] :> Log[a b]
]
```

```
True
```